COMPETING SPIN WAVES AND SUPERCONDUCTING FLUCTUATIONS IN STRONGLY CORRELATED ELECTRON SYSTEMS

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(Received ...)

Abstract

A special diagram technique recently proposed for strongly correlated electron systems is used to study the pecularities of a spin-density-wave (SDW) in competition with superconductivity. This method allows to formulate the Dyson equations for the renormalized electron propagators of the coexisting phases of SDW antiferromagnetism and superconductivity. We find the surprising result that triplet supercondctivity appears provided that we have coexistence of singlet superconductivity and SDW antiferromagnetism. A special ansatz, which takes into account the full Green's functions as well as the dynamical structure of the correlations, is used to establish the equations determining the gap functions and order parameters.

Keywords: Strongly correlated electron systems, Spin-density-wave, Singlet and triplet superconductivity

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1. INTRODUCTION

The spin-density-wave (SDW) state, firstly introduced by Overhauser (Overhauser, 1960, 1962) for the electron gas in crystals, has the wave length $\lambda = 2\pi/|\mathbf{Q}|$ in direction of the vector $\mathbf{Q}$ in the Brillouin zone. These oscillations are commensurate with the crystal structure if the vector $2\mathbf{Q}$ is equal to a vector of the reciprocical lattice. In this paper we discuss strongly correlated elcetron sytems on the basis of the Hubbard model and assume that the strong intra-atomic Coulomb correlations lead to the spontaneous formation of a permanent static SDW existing under certain conditions. The SDW is a special case of collective motions of electron–hole pairs. The broken symmetry is manifested by the transition from the paramagnetic state to the itinerant antiferromagnetic phase. In the case of helicoidal (spiral) polarisation of the SDW, as discussed here, the new phase is characterized by the simultaneous presence of propagators with diagonal spin indices and (anomalous) propagators having off-diagonal spin indices. The transition to the SDW state with broken translational symmetry usually is of second order. This transition is similar to the superconducting transition, which is also discussed in this paper. In the latter case the phase transition is accompanied by a spontanously broken gauge symmetry of the ground state, leading to the appearance of anomalous pairing propagators which do not conserve the number of particles.

The SDW and superconductivity result from the Coulomb interactions of the electrons in the system. In the case of strongly correlated electrons considered here, the strong on-site electron repulsion is (together with the number of electrons per site) is the dominant parameter of the theory. As an elemental model which takes this into account, we use the Hubbard Hamiltonian and the method of broken symmetry in order to discuss the coexistence of a SDW with spiral polarization and superconductivity.

The Hubbard Hamiltonian has the form:

$$H^0 = -\mu \sum_{i\sigma} c^\dagger_{i\sigma} c_{i\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}, \tag{1}$$

$$H^1 = \sum_{ij\sigma} t(j-i) c^\dagger_{j\sigma} c_{i\sigma}. \tag{2}$$

Here c_i and $c_i^\dagger$ are the destruction and creation operators of the electrons at site i, respectively; μ is the chemical potential of the system, $t(j-i)$ is the transfer matrix element and U is the on–site Coulomb repulsion, which is kept in the zero order Hamiltonian.

In order to take into account from the beginning the static SDW, we use the following unitary transformation,

$$\widetilde{H} = \Omega H \Omega^{-1} \tag{3}$$

with the unitary operator Ω defined by

$$\Omega = \prod_i \exp\left(i\theta_i S_i^z\right), \qquad (4)$$

$$\theta_i = (\mathbf{Q}\cdot\mathbf{R}_i), \qquad (5)$$

where S_i^z is the z-component of the electron spin operator at site i. After such a transformation the electron operators $c_{i\sigma}$ and $c^\dagger_{i\sigma}$ take the form ($\sigma = \pm 1$):

$$\tilde{c}_{i,\sigma} = \Omega c_{i,\sigma}\Omega^{-1} = c_{i,\sigma}\exp\left(-i\tfrac{1}{2}\sigma(\mathbf{Q}\cdot\mathbf{R}_i)\right), \qquad (6)$$

$$\tilde{c}^\dagger_{i\sigma} = \Omega c^\dagger_{i\sigma}\Omega^{-1} = c^\dagger_{i,\sigma}\exp\left(i\tfrac{1}{2}\sigma(\mathbf{Q}\cdot\mathbf{R}_i)\right), \qquad (7)$$

This new ground state is characterized by the wave vector $\mathbf{Q}$. The new tunneling matrix element,

$$t_\sigma(\mathbf{R}_j - \mathbf{R}_i) = t(\mathbf{R}_j - \mathbf{R}_i)\cdot\exp\left\{i\tfrac{1}{2}\sigma\left[\mathbf{Q}\cdot(\mathbf{R}_j - \mathbf{R}_i)\right]\right\}, \qquad (8)$$

depends now on the wave vector and on the spin of the tunneling electrons. The Fourier transform of this new matrix element is equal to

$$e_\sigma(\mathbf{k}) = e(\mathbf{k} + \sigma\tfrac{1}{2}\mathbf{Q}), \qquad (9)$$

where $e(\mathbf{k})$ is the Fourier transform of the initial matrix element. This unitary transformation rotates the transversal components of the electron spin operator:

$$\Omega S_i^x\Omega^{-1} = \cos(\mathbf{Q}\cdot\mathbf{R}_i)\cdot S_i^x - \sin(\mathbf{Q}\cdot\mathbf{R}_i)\cdot S_i^y, \qquad (10)$$

$$\Omega S_i^y\Omega^{-1} = \cos(\mathbf{Q}\cdot\mathbf{R}_i)\cdot S_i^x + \sin(\mathbf{Q}\cdot\mathbf{R}_i)\cdot S_i^x. \qquad (11)$$

In this paper we discuss the properties of a SDW having one of the two spiral polarizations. Therefore, in order to obtain non zero values of thermodynamical averages of the transverse components of the spin operator, it is necessary to break the spin conservation law of the initial Hamiltonian. This is realized by adding to the Hamiltonian a source term for the formation of the SDW. In our case this term corresponds to one of the transversal components of the full spin operator of the system. This allows for the formation of anomalous expectation values; after the renormalization of these quantities the source is removed restoring the initial Hamiltonian. This procedure allows to introduce from the beginning the static spin wave corresponding to broken symmetry of the ground state. Let us emphasize that this method does not correspond to the concept of mean field approximation used in the works of (Machida and Matsubara, 1981; Machida, 1982a,b, 1983). Instead we investigate the properties of the renormalized one-particle Matsubara Green's functions for the case of coexisting superconductivity and SDW phases:

$$G_{\sigma\sigma'}(x - x') = -\langle T c_{\mathbf{x}\sigma}(\tau)\bar{c}_{\mathbf{x}'\sigma'}(\tau')U(\beta)\rangle_0^c, \qquad (12)$$

$$F_{\sigma\sigma'}(x - x') = -\langle T c_{\mathbf{x}\sigma}(\tau)c_{\mathbf{x}'\sigma'}(\tau')U(\beta)\rangle_0^c, \qquad (13)$$

$$\bar{F}_{\sigma\sigma'}(x - x') = -\langle T \bar{c}_{\mathbf{x}\sigma}(\tau)\bar{c}_{\mathbf{x}'\sigma'}(\tau')U(\beta)\rangle_0^c. \qquad (14)$$

Here $U(\beta)$ is the evolution operator, $x = (\mathbf{x}, \tau)$ and $c_{\mathbf{x},\sigma}(\tau)$ is the electron operator in the interaction representation.

2. THE SYSTEM OF DYSON EQUATIONS

We use a new diagrammatic method, recently elaborated by us for strongly correlated systems, based on a generalized Wick theorem and a new conception of irreducible Green's functions (Vladimir and Moskalenko, 1990; Vakaru, Vladimir and Moskalenko, 1990; Bogoliubov and Moskalenko, 1991, 1992; Moskalenko, Entel and Digor, 1999; Moskalenko, Entel, Marinaro, Perkins *et al.*, 2000; Moskalenko, Entel, Digor, Marinaro *et al.*, 2003). The irreducible Green's functions describe all spin, charge and pairing fluctuations which are possible in the system. Thus, an approximate knowledge of these quantities will allow a serious discussion of the occurrence of multiple phase transitions and competition between different phases. The infinite sum of these new elements lead to new correlation functions, $Z_{\sigma\sigma'}$, $Y_{\sigma\sigma'}$ and $\bar{Y}_{\sigma\sigma'}$, which are the most essential elements of the theory describing spin, charge and pairing tendencies. The Dyson equations for the delocalized Green's functions, $G_{\sigma\sigma'}$, $F_{\sigma\sigma'}$ and $\bar{F}_{\sigma\sigma'}$, contain these correlation functions together with the electronic energy, $e_\sigma(\mathbf{k}) = e(\mathbf{k} + \sigma\frac{1}{2}\mathbf{Q})$, for different values of spin σ and SDW wave vector $\mathbf{Q}$. The simplest form of these equations can be obtained by using the matrix notation for the full Green's functions $\hat{G}(k)$ $(k = \mathbf{k}, i\omega)$,

$$\hat{G}(k) = \begin{pmatrix} G_{\uparrow\uparrow}(k) & G_{\uparrow\downarrow}(k) & F_{\uparrow\downarrow}(k) & F_{\uparrow\uparrow}(k) \\ G_{\downarrow\uparrow}(k) & G_{\downarrow\downarrow}(k) & F_{\downarrow\downarrow}(k) & F_{\downarrow\uparrow}(k) \\ \bar{F}_{\downarrow\uparrow}(k) & \bar{F}_{\downarrow\downarrow}(k) & -G_{\downarrow\downarrow}(-k) & -G_{\uparrow\downarrow}(-k) \\ \bar{F}_{\uparrow\uparrow}(k) & \bar{F}_{\uparrow\downarrow}(k) & -G_{\downarrow\uparrow}(-k) & -G_{\uparrow\uparrow}(-k) \end{pmatrix} \tag{15}$$

with mass operator $\hat{e}(\mathbf{k})$:

$$\hat{e}(\mathbf{k}) = \begin{pmatrix} e_\uparrow(\mathbf{k}) & o & 0 & 0 \\ 0 & e_\downarrow(\mathbf{k}) & o & o \\ 0 & 0 & -e_\downarrow(-\mathbf{k}) & 0 \\ 0 & 0 & 0 & -e_\uparrow(-\mathbf{k}) \end{pmatrix} \tag{16}$$

and correlation matrix $\hat{Z}(k)$:

$$\hat{Z}(k) = \begin{pmatrix} Z_{\uparrow\uparrow}(k) & Z_{\uparrow\downarrow}(k) & Y_{\uparrow\downarrow}(k) & Y_{\uparrow\uparrow}(k) \\ Z_{\downarrow\uparrow}(k) & Z_{\downarrow\downarrow}(k) & Y_{\downarrow\downarrow}(k) & Y_{\downarrow\uparrow}(k) \\ \bar{Y}_{\downarrow\uparrow}(k) & \bar{Y}_{\downarrow\downarrow}(k) & -Z_{\downarrow\downarrow}(-k) & -Z_{\uparrow\downarrow}(-k) \\ \bar{Y}_{\uparrow\uparrow}(k) & \bar{Y}_{\uparrow\downarrow}(k) & -Z_{\downarrow\uparrow}(-k) & -Z_{\uparrow\uparrow}(-k) \end{pmatrix} \tag{17}$$

leading to the following matrix equation for $\hat{G}(k)$:

$$\hat{G}(k) = \hat{\Lambda}(k)\left[1+\hat{e}(k)\cdot\hat{G}(k)\right], \quad \hat{\Lambda}(k)=\hat{G}^{(0)}+\hat{Z}(k), \tag{18}$$

$$G_{\sigma}^{(0)} = \frac{1}{Z_0}\left[\frac{\left(e^{-\beta E_0}+e^{-\beta E_\sigma}\right)}{\lambda_\sigma(i\omega)}+\frac{\left(e^{-\beta E_{-\sigma}}+e^{-\beta E_2}\right)}{\bar{\lambda}_\sigma(i\omega)}\right], \tag{19}$$

$$\lambda_\sigma = i\omega+E_0-E_\sigma, \tag{20}$$

$$\bar{\lambda}_{-\sigma} = i\omega+E_{-\sigma}-E_2. \tag{21}$$

Here E_0, E_σ and E_2 are the local ion energies for the empty, one spin and double spin states, correspondingly. As remarked before, the system of Dyson equations allow for the additional appearance of triplet superconductivity in the presence of singlet superconductivity and a spiral SDW state (or the appearance of a SDW, if singlet and triplet superconductivity coexist).

In order to close the equations of motion for the full Green's functions it is necessary to add to them the corresponding equations for the correlation functions. Since the Dyson equations for these functions do not exist we must use appropriate approximations, which are based on the procedure of summing the most important diagrams. Here we use the local approximation in coordinate space for these quantities which is obtained by summing one class of diagrams containing the simplest two-particle irreducible Green's functions $G_2^{(0)\,irr}$. These diagrams give the main contribution in this theory.

$$\begin{aligned} Z_{\sigma\sigma}(i\omega) = & -\frac{1}{\beta N}\sum_{\mathbf{k}}\sum_{\omega_1}\Big\{\widetilde{G}_2^{(0)\,irr}\left[\sigma,i\omega;\sigma,i\omega_1\mid\sigma,i\omega_1;\sigma,i\omega\right] \\ & \times e_\sigma^2(\mathbf{k})\,G_{\sigma\sigma}(\mathbf{k}\mid i\omega_1)+\widetilde{G}_2^{(0)\,irr}\left[\sigma,i\omega;\bar{\sigma},i\omega_1\mid\bar{\sigma},i\omega_1;\sigma,i\omega\right] \\ & \times e_{\bar{\sigma}}^2(\mathbf{k})\,G_{\bar{\sigma}\bar{\sigma}}(\mathbf{k}\mid i\omega_1)\Big\}, \end{aligned} \tag{22}$$

$$\begin{aligned} Z_{\sigma\bar{\sigma}}(i\omega) = & -\frac{1}{\beta N}\sum_{\mathbf{k}}\sum_{\omega_1}\widetilde{G}_2^{(0)\,irr}\left[\sigma,i\omega;\bar{\sigma},i\omega_1\mid\sigma,i\omega_1;\bar{\sigma},i\omega\right] \\ & \times e_\sigma(\mathbf{k})e_{\bar{\sigma}}(\mathbf{k})G_{\sigma\bar{\sigma}}(\mathbf{k}\mid i\omega_1), \end{aligned} \tag{23}$$

$$\begin{aligned} \bar{Y}_{\bar{\sigma}\sigma}(i\omega) = & -\frac{1}{\beta N}\sum_{\mathbf{k}}\sum_{\omega_1}\widetilde{G}_2^{(0)\,irr}\left[\sigma,i\omega_1;\bar{\sigma},-i\omega_1\mid\sigma,i\omega;\bar{\sigma},-i\omega\right] \\ & \times e_\sigma(\mathbf{k})\,e_{\bar{\sigma}}(-\mathbf{k})\,\bar{F}_{\bar{\sigma}\sigma}(\mathbf{k}\mid i\omega_1), \end{aligned} \tag{24}$$

$$\begin{aligned} \bar{Y}_{\sigma\sigma}(i\omega) = & -\frac{1}{2\beta N}\sum_{\mathbf{k}}\sum_{\omega_1}\widetilde{G}_2^{(0)\,irr}\left[\sigma,i\omega_1;\sigma,-i\omega_1\mid\sigma,i\omega;\sigma,-i\omega\right] \\ & \times e_\sigma(\mathbf{k})\,e_{\sigma}(-\mathbf{k})\,\bar{F}_{\sigma\sigma}(\mathbf{k}\mid i\omega_1). \end{aligned} \tag{25}$$

The kernels of these equations are the simplest irreducible two-particle Green's functions,

$$
\begin{aligned}
&G_2^{(0)\,irr}\left[\sigma_1, i\omega_1; \sigma_2, i\omega_2 \mid \sigma_3, i\omega_3; \sigma_4, i\omega_4\right] \\
&= G_2^{(0)}\left[\sigma_1, i\omega_1; \sigma_2, i\omega_2 \mid \sigma_3, i\omega_3; \sigma_4, i\omega_4\right] \\
&+ \beta^2\left[\delta_{\omega_1,\omega_3}\delta_{\omega_2,\omega_4}\delta_{\sigma_1,\sigma_3}\delta_{\sigma_2,\sigma_4} - \delta_{\omega_1,\omega_4}\delta_{\omega_2,\omega_3}\delta_{\sigma_1,\sigma_4}\delta_{\sigma_2,\sigma_3}\right] \\
&\times G_{\sigma_1}^{(0)}(i\omega_1) G_{\sigma_2}^{(0)}(i\omega_2),
\end{aligned} \tag{26}
$$

which obey spin and frequency conservation. These equations for the correlation functions together with the Dyson equations lead to a closed system of equations, the numerical investigation of which has yet to be undertaken.

3. ITINERANT ANTIFERROMAGNETISM

As the simplest demonstration of the theory presented here we shall discuss antiferromagnetism of strongly correlated electrons in the presence of a SDW with spiral polarization. In this case we set in the previous equations the superconducting Green's functions equal to zero leading to the following more simple expresions:

$$
G_{\sigma\sigma}(k) = \frac{\Lambda_{\sigma\sigma}(k)\left(1 - \Lambda_{\bar{\sigma}\bar{\sigma}}(k) e_{\bar{\sigma}}(\mathbf{k})\right) + e_{\bar{\sigma}}(\mathbf{k}) Z_{\sigma\bar{\sigma}}(k) Z_{\bar{\sigma}\sigma}(k)}{D^{AF}(\sigma \mid k)}, \tag{27}
$$

$$
G_{\bar{\sigma}\sigma}(k) = \frac{Z_{\bar{\sigma}\sigma}(k)}{D^{AF}(\sigma \mid k)}, \tag{28}
$$

$$
\Lambda_{\sigma\sigma}(k) = G_\sigma^{(0)}(k) + Z_{\sigma\sigma}(k), \tag{29}
$$

$$
\begin{aligned}
D^{AF}(\sigma \mid k) &= \left[1 - \Lambda_{\sigma\sigma}(k) e_\sigma(\mathbf{k})\right]\left[1 - \Lambda_{\bar{\sigma}\bar{\sigma}} e_{\bar{\sigma}}(\mathbf{k})\right] \\
&- e_\sigma(\mathbf{k}) e_{\bar{\sigma}}(\mathbf{k}) Z_{\sigma\bar{\sigma}}(k) Z_{\bar{\sigma}\sigma}(k).
\end{aligned} \tag{30}
$$

The kernels of these equations take a more simple form in case of half-filling for which ($\mu = U/2$) holds. This leads to

$$
\widetilde{G}_2^{(0)\,irr}\left[\sigma, i\omega; \sigma, i\omega_1 \mid \sigma, i\omega_1; \sigma, i\omega\right] = \beta\mu^2 \frac{\left[\delta_{\omega,\omega_1} - 1\right]}{\left[\omega^2 + \mu^2\right]\left[\omega_1^2 + \mu^2\right]}, \tag{31}
$$

$$
\begin{aligned}
&\widetilde{G}_2^{(0)\,irr}\left[\sigma, i\omega; \bar{\sigma}, i\omega_1 \mid \bar{\sigma}, i\omega_1; \sigma, i\omega\right] = \frac{2\beta\mu^2}{Z_0} \frac{\left(e^{-\beta E_0} - e^{-\beta E_\sigma}\right)}{\left[\omega^2 + \mu^2\right]\left[\omega_1^2 + \mu^2\right]} \\
&- \frac{4\beta\mu^2 \delta_{\omega,\omega_1} e^{-\beta E_\sigma}}{Z_0\left[\omega^2 + \mu^2\right]^2} + \frac{4\beta\mu^2 \delta_{\omega,-\omega_1} e^{-\beta E_0}}{Z_0\left[\omega^2 + \mu^2\right]^2} - \frac{2\mu}{\left[\omega^2 + \mu^2\right]\left[\omega_1^2 + \mu^2\right]} \\
&+ \frac{4\mu^3\left(\omega^2 + \omega_1^2 + 2\mu^2\right)}{\left(\omega^2 + \mu^2\right)^2\left(\omega_1^2 + \mu^2\right)^2},
\end{aligned} \tag{32}
$$

$$\widetilde{G}_2^{(0)\,irr}\left[\sigma,i\omega;\bar{\sigma},i\omega_1 \mid \sigma,i\omega_1;\bar{\sigma},i\omega\right] = -\frac{2\beta\mu^2\delta_{\omega,\omega_1}\left(e^{-\beta E_0}-e^{-\beta E_\sigma}\right)}{Z_0\left[\omega^2+\mu^2\right]^2}$$
$$-\frac{4\beta\mu^2\delta_{\omega,-\omega_1}e^{-\beta E_0}}{Z_0\left[\omega^2+\mu^2\right]^2}+\frac{4\beta\mu^2 e^{-\beta E_\sigma}}{Z_0\left(\omega^2+\mu^2\right)\left(\omega_1^2+\mu^2\right)}$$
$$+\frac{2\mu}{\left(\omega^2+\mu^2\right)\left(\omega_1^2+\mu^2\right)}-\frac{4\mu^3\left(\omega^2+\omega_1^2+2\mu^2\right)}{\left(\omega^2+\mu^2\right)^2\left(\omega_1^2+\mu^2\right)^2}. \tag{33}$$

In order to solve the resulting equations, we assume that the local correlation functions have some special structure relying on three new functions $\zeta(i\omega), \chi(i\omega)$ and $\Delta(i\omega)$ which have to be determined from

$$\Lambda_{\sigma\sigma}(i\omega) = \frac{i\widetilde{\omega}+\sigma\chi(i\omega)}{\left[(i\widetilde{\omega})^2-E_1^2\right]}, \tag{34}$$

$$Z_{\sigma\bar{\sigma}}(i\omega) = Z_{\bar{\sigma}\sigma}(i\omega) = \frac{\Delta(i\omega)}{\left[(i\widetilde{\omega})^2-E_1^2\right]}, \tag{35}$$

$$E_1^2 = \Delta^2(i\omega)+\chi^2(i\omega), \tag{36}$$

$$\widetilde{\omega} = \omega\zeta(i\omega). \tag{37}$$

Here the function $\zeta(i\omega)$ renormalizes the Matsubara frequency, the function χ determines the renormalized excitation energy and Δ is the energy gap and order parameter of the SDW state. The Dyson equations permit us then to obtain the corresponding Dyson equations for the delocalized Green's functions:

$$G_{\sigma\sigma}(k) = \frac{i\widetilde{\omega}+\sigma\chi(i\omega)-e_{\bar{\sigma}}(\vec{k})}{\left(i\widetilde{\omega}-E_+(k)\right)\left(i\widetilde{\omega}-E_-(k)\right)}, \tag{38}$$

$$G_{\sigma\bar{\sigma}}(k) = G_{\bar{\sigma}\sigma}(k) = \frac{\Delta(i\omega)}{\left(i\widetilde{\omega}-E_+(k)\right)\left(i\widetilde{\omega}-E_-(k)\right)}, \tag{39}$$

$$D^{AF}(k) = \frac{\left(i\widetilde{\omega}-E_+(k)\right)\left(i\widetilde{\omega}-E_-(k)\right)}{\left[(i\widetilde{\omega})^2-E_1^2\right]}; \tag{40}$$

$$E_\pm(k) = \frac{e_\sigma(\vec{k})+e_{\bar{\sigma}}(\vec{k})}{2}\pm E(k), \tag{41}$$

$$E(k) = \left[\Delta^2(i\omega)+\left(\sigma\chi(i\omega)+\frac{e_\sigma(\vec{k})-e_{\bar{\sigma}}(\vec{k})}{2}\right)^2\right]^{\frac{1}{2}}. \tag{42}$$

If in addition we assume nesting of the form $e_{\bar{\sigma}}(\vec{k}) = -e_\sigma(\vec{k})$ together with half-filling, the equations have the form:

$$
\begin{aligned}
\frac{i\widetilde{\omega}}{(i\widetilde{\omega})^2 - E_1^2} &= \frac{i\omega}{(i\omega)^2 - \mu^2} - \frac{1}{\beta N}\sum_{\vec{k}}\sum_{\omega_1} \frac{i\widetilde{\omega_1} e_\sigma^2(\vec{k})}{\left[(i\widetilde{\omega}_1)^2 - E^2(\vec{k}|i\omega_1)\right]} \\
&\times \left[\widetilde{G}_2^{(0)\,irr}\left[\sigma, i\omega; \sigma, i\omega_1 \mid \sigma, i\omega_1; \sigma, i\omega\right]\right. \\
&+ \left.\widetilde{G}_2^{(0)\,irr}\left[\sigma, i\omega; \bar{\sigma}, i\omega_1 \mid \bar{\sigma}, i\omega_1; \sigma, i\omega\right]\right], \qquad (43)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\chi(i\omega)}{(i\widetilde{\omega})^2 - E_1^2} &= -\frac{1}{\beta N}\sum_{\vec{k}}\sum_{\omega_1} \frac{\left(\chi(i\omega_1) + \sigma e_\sigma(\vec{k})\right) e_\sigma^2(\vec{k})}{\left[(i\widetilde{\omega}_1)^2 - E^2(\vec{k} \mid i\omega_1)\right]} \\
&\times \left[\widetilde{G}_2^{(0)\,irr}\left[\sigma, i\omega; \sigma, i\omega_1 \mid \sigma, i\omega_1; \sigma, i\omega\right]\right. \\
&- \left.\widetilde{G}_2^{(0)\,irr}\left[\sigma, i\omega; \bar{\sigma}, i\omega_1 \mid \bar{\sigma}, i\omega_1; \sigma, i\omega\right]\right], \qquad (44)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\Delta(i\omega)}{(i\widetilde{\omega})^2 - E_1^2} &= \frac{1}{\beta N}\sum_{\vec{k}}\sum_{\omega_1} \frac{\Delta(\omega_1) e_\sigma^2(\vec{k})}{\left[(i\widetilde{\omega}_1)^2 - E^2(\vec{k} \mid i\omega_1)\right]} \\
&\times \left[\widetilde{G}_2^{(0)\,irr}\left[\sigma, i\omega; \bar{\sigma}, i\omega_1 \mid \sigma, i\omega_1; \sigma, i\bar{\omega}\right]\right]. \qquad (45)
\end{aligned}
$$

The critical temperature at which the phase transition to the SDW state takes place is determined from last equation by linearizing it with respect to the order parameter.

4. SUMMARY AND CONCLUSIONS

The main equations for the renormalized one-particle Green's and correlation functions of strongly correlated electron systems have been derived for the mixed phases of coexisting SDW with spiral polarization and superconductivity. As model Hamiltonian we have used the single-band Hubbard model; in the corresponding diagrammatic expansion recently proposed for the description of strongly correlated electron systems the electron transfer term is used as the perturbative term. As new elements of the theory correlation functions appear, which contain the most important spin, charge and pairing fluctuations. The correlation functions together with the full Green's functions are the main elements of the new diagrammatic approach. We have then established the exact Dyson equations for the delocalized Green's functions. With respect to the equations for the correlation functions we have used simple, but in the sense of retaining the most important contributions optimum approximations. The influence of a spiral polarized SDW on the appearance of triplet superconductivity and vice versa has been shown. For the special case of half-filling and perfect nesting condition the equations of motion for the itinerant antiferromagnetic state take on their simplest form, however, a numerical investigation has yet to be undertaken.

Acknowledgements

The authors would like to thank the Steering Committee of the Heisenberg-Landau Program for the Grant which supports this work. P. Entel would like to thank the Bogoliubov Laboratory of Theoretical Physics of JINR (Dubna) for its hospitality. V. A. Moskalenko thanks the theory group of the Institute of Physics of the University of Duisburg-Essen for its hospitality and the SFB 445 for financial support. V. A. Moskalenko and D. F. Digor thank the High Department of Science and Technology of the Moldova Republic Government for a Grant.

References

Bogoliubov, N. N. and V. A. Moskalenko (1991). On question of the superconductivity in the Hubbard model. *Theor. Math. Phys.,* **86**, 16.

Bogoliubov, N. N. and V. A. Moskalenko (1992). Superconductivity of the Hubbard model with deviation from the half filling. *Theor. Math. Phys.,* **92**, 182.

Machida, K. (1982a). Energy gap structure in spin density wave superconductors. *Solid State Commun.,* **41**, 217.

Machida, K. (1982b). Spin density wave and superconductivity in highly anisotropic materials. III Energy gap structure and non-magnetic impurities effects. *J. Phys. Soc. Jpn.,* **51**, 1420.

Machida, K. (1983). Spin density wave and superconductivity in highly anisotropic materials. IV Electromagnetic properties of the coexistence phase. *J. Phys. Soc. Jpn.,* **52**, 1333.

Machida, K. and T. Matsubara (1981). Spin density wave and superconductivity in highly anisotropic materials. II Detailed study of phase transitions. *J. Phys. Soc. Jpn.,* **50**, 3231.

Moskalenko, V. A., P. Entel and D. F. Digor (1999). Strong interaction of correlated electrons with phonons: A diagrammatic approach. *Phys. Rev. B.,* **59**, 619.

Moskalenko, V. A., P. Entel, D. F. Digor, M. Marinaro *et al.* (2003). Models for strongly interacting fermion systems. In J. K. Srivastava and S. M. Rao, editors, *Models and Methods of High-T_c Superconductivity. Some Frontal Aspects*, volume 242 of *Horizons in World Physics*, chapter 6, 201. Nova Science, New York.

Moskalenko, V. A., P. Entel, M. Marinaro, N. B. Perkins *et al.* (2000). Hopping perturbation treatment of the periodic Anderson model around the atomic limit. *Phys. Rev. B,* **63**, 245119.

Overhauser, A. W. (1960). Giant spin density waves. *Phys. Rev. Lett.,* **4**, 462.

Overhauser, A. W. (1962). Spin density waves in an electron gas. *Phys. Rev.,* **128**, 1437.

Vakaru, S. I., M. I. Vladimir and V. A. Moskalenko (1990). Diagram technique for the Hubbard model. II Metal-insulator transition. *Theor. Math. Phys.,* **85**, 248.

Vladimir, M. I. and V. A. Moskalenko (1990). Diagram technique for the Hubbard model. *Theor. Math. Phys.,* **82**, 428.